From quantum tunneling in a topology-changing fermionic bath to topological quantum superpositions

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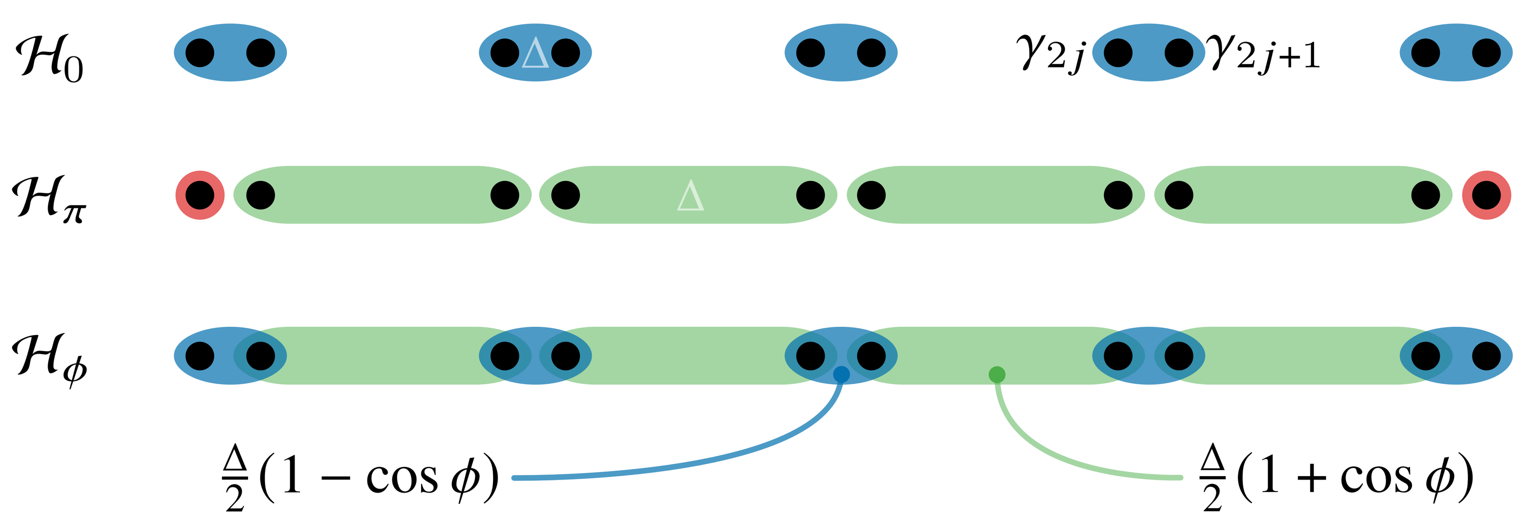
• In planar Josephson Majorana systems, the superconducting phase difference ϕ determines the topology [1 - 4].
• Coulomb effects: ϕ becomes a quantum variable.
⇒ Prospect of ***quantum tunneling*** between two topologically distinct phases.
• Tunneling is suppressed by the many-fermion bath changing its topology, which we quantify using instanton methods.
• Suppression is ***incomplete*** - can in principle generate a ***superposition of Majoranas and no Majoranas***.

Effective Model

• Minimal model:

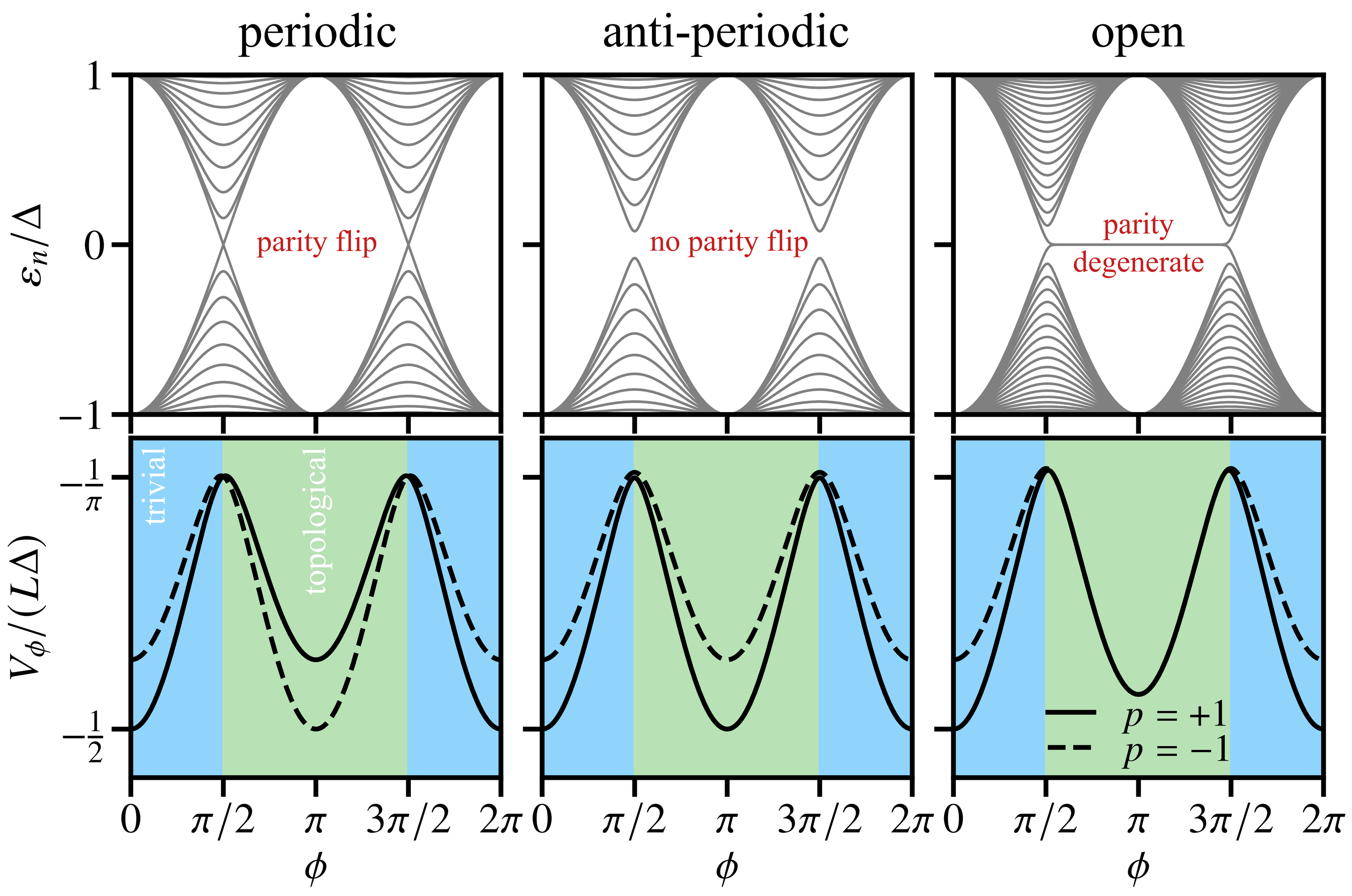
$$\mathcal{H}_\phi = \tfrac{1}{2}(1-\cos\phi)\mathcal{H}_0 + \tfrac{1}{2}(1+\cos\phi)\mathcal{H}_\pi$$

Fig. A



• Ground state energy has two equal depth minima (1 topological, 1 trivial phase):

Fig. B



• Add charging energy term due to intrinsic capacitance, reminiscent of Cooper-pair box with canonically conjugate charge and phase: think of ϕ as *position*, and N as *momentum*.

$$H = \underbrace{E_C(N-N_g)^2}_{\text{"Kinetic Energy"}} + \underbrace{\tfrac{1}{2}\textstyle\sum_k \Psi_k^\dagger \mathcal{H}_\phi(k)\Psi_k}_{\text{"Potential Energy"}}$$

$$[\phi, N] = 2i$$

• Effective model captures crucial ingredient that ϕ is coupled to a many-fermion system.
• For isolated potential wells, state of ϕ would be harmonic-oscillator-like, with fermions either topological or trivial.
• Wells ***not*** isolated ⇒ tunneling amplitude $t^{(\mathrm{f})}_{0\to\pi} \neq 0$ between topologically distinct minima!
• Evaluating $t^{(\mathrm{f})}_{0\to\pi}$ difficult: fermionic part of the wavefunction changes with ϕ.

Instanton Method [5]

$$Z = \int \mathcal{D}\phi\mathcal{D}c\mathcal{D}\bar{c}\, e^{-S}$$

$$S = \frac{1}{2}\int_0^\beta d\tau \left[\frac{1}{8E_C}(\partial_\tau\phi_\tau)^2 + iN_g(\partial_\tau\phi_\tau) + \underbrace{\bar{\Psi}^T(\partial_\tau + \mathcal{H}_{\phi_\tau})\Psi}_{\text{Integrate out}}\right]$$

• Z dominated by saddle points of the action, where ϕ_τ is a sequence of π phase slips, known as *instantons*.
• Integrating out the fermions generates an effective (non-local) potential felt by ϕ.

$$U_\mathrm{f}[\phi_\tau] \equiv -\tfrac{1}{2}\log\det\left[\partial_\tau + \mathcal{H}_{\phi_\tau}\right] \qquad \{\sigma_1, \mathcal{H}_\phi\} = 0$$

$$\det\left[\partial_\tau + \mathcal{H}_{\phi_\tau}\right] = \det\left[i\sigma_1\partial_\tau + i\sigma_1\mathcal{H}_{\phi_\tau}\right] \equiv \det\left[\tilde{\mathcal{H}}_{\phi_\tau}\right]$$

• The product of energies of $\tilde{\mathcal{H}}_{\phi_\tau}$ with τ as an extra spatial dimension.
⇒ Each instanton is a topological phase boundary of a 2D Class D superconductor, hosting sub-gap chiral modes.
• These edge states at the core of fermionic tunneling suppression.

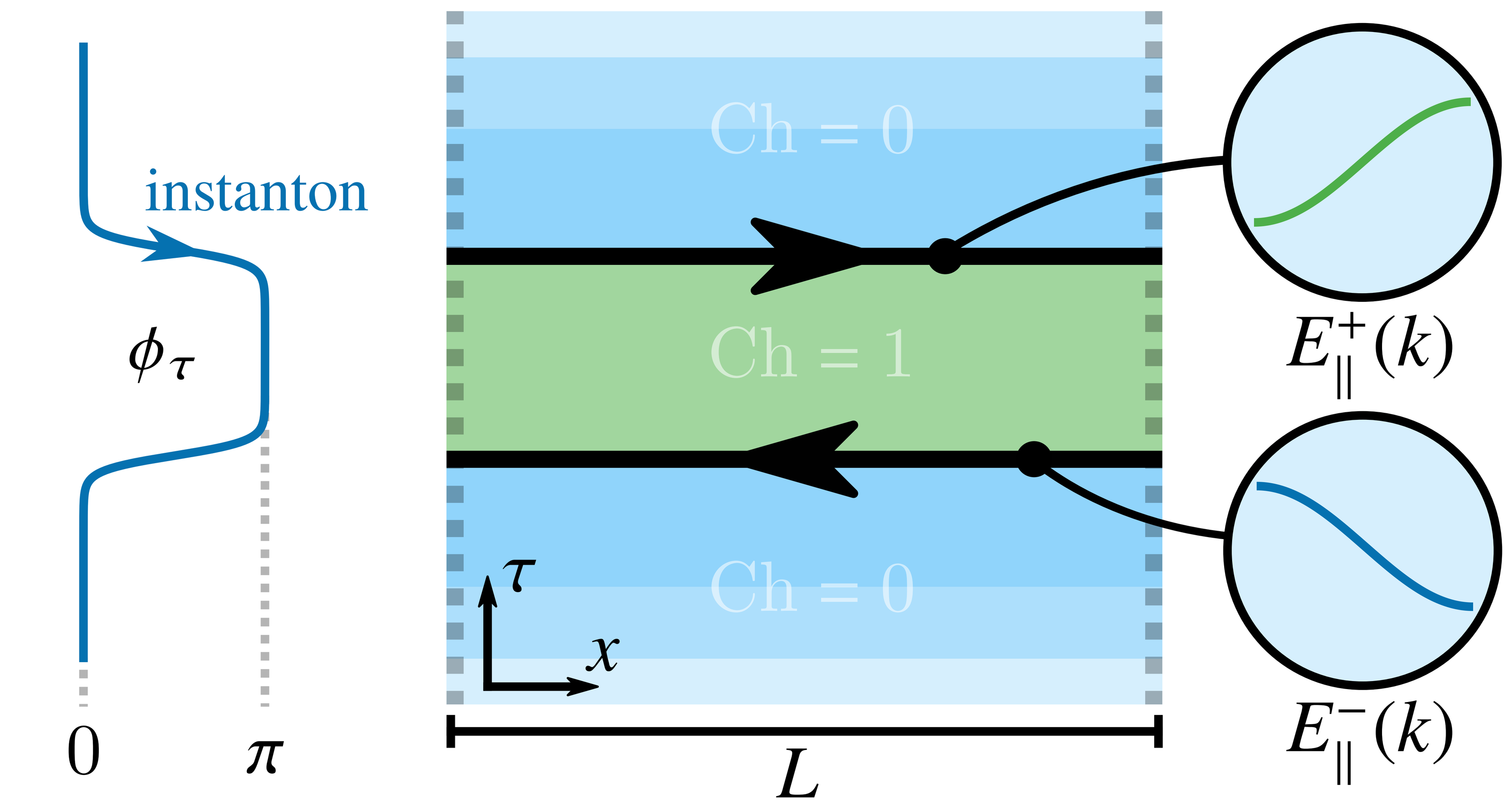


Fig. C

• The L sub-gap states for each instanton are guaranteed by topology: even a perfectly sharp instanton has a potential energy action cost $\propto L$, unlike a scalar potential.

$$\frac{\det\left[\partial_\tau + \mathcal{H}_{\phi_{2\mathrm{i}}}\right]}{\det\left[\partial_\tau + \mathcal{H}_{\phi_{0\mathrm{i}}}\right]} = \left[\prod_{n=0}^{L-1}\sin(k_n/2)\right]^2 = 2^{-2(L-1)}$$

• Sharp instanton picture captures essence of more refined time-ordered exponential method.

Results

• Topology-changing fermions suppress the tunneling amplitude compared to a naïve Josephson potential V_ϕ in Fig. B.
• Sharp instanton limit gives upper bound on suppression:

$$t^{(\mathrm{f})}_{0\to\pi} \geq e^{-\frac{1}{2}(L-1)\log 2}\, t^{(\mathrm{n})}_{0\to\pi}$$

• Suppression is weaker for smaller E_C because instantons have a longer timescale, approaching adiabaticity (i.e., naïve V_ϕ physics).
• Fermionic topology never blocks tunneling completely.
⇒ Superposition of two seemingly mutually exclusive possibilities: Majoranas being both present and absent!

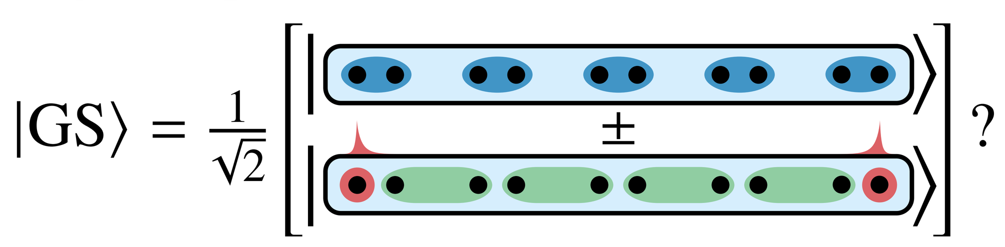


Fig. D

• Requires L short enough to have tunneling, but long enough relative to the width of Majorana bound states (set independently from $t^{(\mathrm{f})}_{0\to\pi}$) to suppress Majoranas splitting.

References

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[5] Coleman. *Aspects of Symmetry.* Selected Erice Lectures (1985)